प्रश्नावली 15.1

Type I. हल करें :

1. $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

2. $\log \frac{dy}{dx} = ax + by$ [संकेत : $\frac{dy}{dx} = e^{ax+by}$]

3. $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

4. $y\sqrt{1-x^2}\, dy + x\sqrt{1-y^2}\, dx = 0$ [UBT]

5. $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

6. $(x^2 - yx^2)\, dy + (y^2 + xy^2)\, dx = 0$

7. $(xy^2 + x)\, dx + (yx^2 + y)\, dy = 0$

8. $\sin x \cos y \frac{dy}{dx} + \cos x \sin y = 0$ [संकेत : $\frac{\cos y}{\sin y} dy + \frac{\cos x}{\sin x} dx = 0$]

9. $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$

Type II. हल करें :

10. $\frac{dy}{dx} = \frac{x+y+3}{2x+2y+1}$

11. $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

उत्तरमाला

1. $y - x = c(1 + xy)$
2. $bx^{ax} + ae - by = c$
3. $(1 + e^y)\sin x = c$
4. $\sqrt{1-y^2} + \sqrt{1-x^2} = c$
5. $\tan x \tan y = c$
6. $\log \frac{x}{y} - \left(\frac{1}{x} + \frac{1}{y}\right) = c$
7. $(x^2 + 1)(y^2 + 1) = c$
8. $\sin x \sin y = c$
9. $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$
10. $x + c = \frac{2}{3}(x+y) - \frac{5}{9}\log(3x + 3y + 4)$
11. $\log\left(1 + \tan\frac{x+y}{2}\right) = x + c$

Type III. समघाती अवकल समीकरण (Homogeneous Differential Equations)

यदि दिया गया प्रथम घात तथा प्रथम क्रम का अवकल समीकरण

$\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ के रूप का हो, जहाँ $f(x, y)$ तथा $g(x, y)$ में प्रत्येक पद समान घात (degree) का

15.1

(1) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

or $\frac{dy}{1+y^2} - \frac{dx}{1+x^2} = 0$

or $\int \frac{dy}{1+y^2} - \int \frac{dx}{1+x^2} = \tan^{-1} c$

or $\tan^{-1} y - \tan^{-1} x = \tan^{-1} c$

or $\tan^{-1}\frac{(y-x)}{1+xy} = \tan^{-1} c$ or $\frac{y-x}{1+xy} = c$

$y-x = c(1+xy)$ Ans.

(2) $\log \frac{dy}{dx} = ax+by$

$\frac{dy}{dx} = e^{ax+by} = e^{ax} \cdot e^{by}$

or $\frac{dy}{e^{by}} = e^{ax} \cdot dx$

or $e^{-by}\, dy = e^{ax}\, dx$

or $e^{ax}\, dx - e^{-by}\, dy = 0$

or $\int e^{ax}\, dx - \int e^{-by}\, dy = c$

or $\frac{e^{ax}}{a} + \frac{e^{-by}}{b} = c$

or $be^{ax} + ae^{-by} = ab$ Ans

③. $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$

$\Rightarrow \int \frac{\cos x}{\sin x}\,dx + \int \frac{e^y}{e^y+1}\,dy = \log c$

$\Rightarrow \log \sin x + \log(e^y+1) = \log c$

$\Rightarrow \log \sin x\,(e^y+1) = \log c$

$\Rightarrow \sin x\,(e^y+1) = c$ Ans

④. $y\sqrt{1-x^2}\,dy + x\sqrt{1-y^2}\,dx = 0$

$\Rightarrow \frac{y\,dy}{\sqrt{1-y^2}} + \frac{x\,dx}{\sqrt{1-x^2}} = 0$

$\Rightarrow -\frac{1}{2}\int \frac{(-2y)\,dy}{\sqrt{1-y^2}} - \frac{1}{2}\int \frac{(-2x)\,dx}{\sqrt{1-x^2}} = c'$

$\Rightarrow \int \frac{(-2y)\,dy}{\sqrt{1-y^2}} + \int \frac{(-2x)\,dx}{\sqrt{1-x^2}} = -2c'$

$\Rightarrow \sqrt{1-y^2} + \sqrt{1-x^2} = c$ Ans.

⑤. $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\Rightarrow \frac{\sec^2 x\,dx}{\tan x} + \frac{\sec^2 y\,dy}{\tan y} = 0$

$\Rightarrow \int \frac{\sec^2 x\,dx}{\tan x} + \int \frac{\sec^2 y\,dy}{\tan y} = \log c$

$\Rightarrow \log \tan x + \log \tan y = \log c$

$\Rightarrow \log(\tan x \cdot \tan y) = \log c$

$\Rightarrow \tan x \cdot \tan y = c$ Ans.

⑥ $(x^2 - yx^2)\,dy + (y^2 + xy^2)\,dx = 0$

$\Rightarrow x^2(1-y)\,dy + y^2(1+x)\,dx = 0$

$\Rightarrow \frac{(1-y)\,dy}{y^2} + \frac{(1+x)}{x^2}\,dx = 0$

$\Rightarrow \left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \left(\frac{1}{x^2} + \frac{1}{x}\right)dx = 0$

$\Rightarrow \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int\left(\frac{1}{x^2} + \frac{1}{x}\right)dx = c$

$\Rightarrow -\frac{1}{y} - \log y - \frac{1}{x} + \log x = c$

$\Rightarrow \log x - \log y - \left(\frac{1}{x} + \frac{1}{y}\right) = c$

$\Rightarrow \log(x/y) - \left(\frac{1}{x} + \frac{1}{y}\right) = c$ Ans.

⑦. $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$

$\Rightarrow x(1+y^2)\,dx + y(1+x^2)\,dy = 0$

$\Rightarrow \frac{x\,dx}{1+x^2} + \frac{y\,dy}{1+y^2} = 0$

$\Rightarrow \frac{1}{2}\int\frac{2x\,dx}{1+x^2} + \frac{1}{2}\int\frac{2y\,dy}{1+y^2} = \log c$

$\Rightarrow \frac{2x\,dx}{2(1+x^2)} + \frac{2y\,dy}{2(1+y^2)} = 0$

$\Rightarrow \frac{2x\,dx}{1+x^2} + \frac{2y\,dy}{1+y^2} = 0$

$\Rightarrow \int\frac{2x\,dx}{1+x^2} + \int\frac{2y\,dy}{1+y^2} = \log c$

$\Rightarrow \log(1+x^2) + \log(1+y^2) = \log c$

$\Rightarrow \log(1+x^2)(1+y^2) = \log c \;\Big|\; (1+x^2)(1+y^2) = c$ Ans

(8). $\sin x \cos y \frac{dy}{dx} + \cos x \sin y = 0$

$\Rightarrow \int \frac{\cos y\, dy}{\sin y} + \int \frac{\cos x\, dx}{\sin x} = \log c$

$\Rightarrow \log \sin y + \log \sin x = \log c$

$\Rightarrow \log (\sin x . \sin y) = \log c$

$\sin x . \sin y = c$ Ans

(9). $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$

$\frac{dy}{dx} = (e^{3x} + x^2) e^{-2y}$

$\Rightarrow \int e^{2y} dy = \int (e^{3x} + x^2) dx + c$

$\Rightarrow \frac{1}{2} e^{2y} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$ Ans.

(10). $\frac{dy}{dx} = \frac{x+y+3}{2x+2y+1}$

put $x+y = v$

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \frac{dv}{dx} - 1 = \frac{v+3}{2v+1}$

$\Rightarrow \frac{dv}{dx} = \frac{v+3}{2v+1} + 1 = \frac{v+3+2v+1}{2v+1}$

$\Rightarrow \frac{(2v+1)}{3v+4} dv = dx$

$\Rightarrow \int \frac{(2v+1)}{3v+4} dv = \int dx + c$

$\Rightarrow \int \frac{\frac{2}{3}(3v+4) - \frac{5}{3}}{3v+4} dv = x + c$

$$\frac{2}{3}\int 1\,dv - \frac{5}{3}\int \frac{dv}{3v+4} = x+c$$

$$\frac{2}{3}v - \frac{5}{9}\log(3v+4) = x+c$$

$$\frac{2}{3}(x+y) - \frac{5}{9}\log(3x+3y+4) = x+c$$ Ans

(11) $$\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$$

put $x+y = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} - 1 = \sin v + \cos v$$

$$\frac{dv}{dx} = \sin v + \cos v + 1$$

$$\frac{dv}{1+\sin v+\cos v} = dx$$

$$\frac{dv}{1+2\sin v/2\cos v/2+2\cos^2 v/2-1} = dx$$

$$\frac{dv}{2\cos v/2(\sin v/2+\cos v/2)} = dx$$

$$\frac{dv}{2\cos^2 v/2\left(\frac{\sin v/2+\cos v/2}{\cos v/2}\right)} = dx$$

$$\frac{1}{2}\int \frac{\sec^2 v/2\,dv}{1+\tan v/2} = \int dx + c$$

put $1+\tan v/2 = t$

$\sec^2(v/2)dv = 2dt$

$$\int \frac{dt}{t} = x+c$$

$$\log t = x+c$$

$$\log\left(1+\tan\frac{v}{2}\right) = x+c$$

$$\log\left(1+\tan\frac{x+y}{2}\right) = x+c$$ Ans